COLUMBIA COLLEGE
MID-YEAR EXAMINATION, FEBRUARY 3, 1910
Astronomy 1

Answer three questions only in each numbered group

1, *a*. Define: celestial sphere, declination, hour-angle.
1, *b*. Describe the ecliptic circle and explain why we always see the sun in that circle.
1, *c*. What visible phenomena are produced by the earth's axial rotation?
1, *d*. Prove that the altitude of the celestial pole is everywhere equal to the latitude.
2, *a*. Explain sidereal and solar time.
2, *b*. Why does the vernal equinox occur on or about March 21?
2, *c*. Explain the reason for time-differences between different places on the earth.
2, *d*. In an ordinary horizontal sundial, what is the angle of elevation of the gnomon, and why?
3, *a*. If a small round steel ball is dropped from a tower, will it reach the earth at a point directly under the point from which the ball was allowed to fall?
3, *b*. If not, where will it reach the earth, and why?
3, *c*. How is the length of the earth's radius determined?
3, *d*. What is the "torsional constant" and how is it determined for any given torsion balance?
4, *a*. Why is summer hotter than winter?
4, *b*. In the northern hemisphere, is summer longer or shorter than winter? Why?
4, *c*. Explain tropical and sidereal years.
4, *d*. Explain the supposed relation between the age of the Great Pyramid in Egypt and the precession of the equinoxes.
5, *a*. Explain the aberration of light.
5, *b*. What are the four constituent parts of a date?
5, *c*. What is the leap-year rule in the Gregorian calendar?
5, *d*. How does the apparent angular velocity of the moon on the sky compare with the sun's, and why?
6, *a*. How is the moon's distance from the earth ascertained.
6, *b*. Explain two lunar librations.
6, *c*. What are occultations, and how are they used to determine terrestrial longitudes?
6, *d*. Demonstrate Kepler's law of areas under the action of a central force.
7, *a*. Define sidereal period of a planet,
Synodic period of a planet,
Conjunction.
7, *b*. Derive formula for computing the sidereal period from the synodic period.
7, *c*. Explain the connection between the visibility of a planet and its synodic motion.
7, *d*. Why does the synodic period approach 365 days as a limit for the outermost planets of the solar system?

THE DEFINITION OF FORCE

To THE EDITOR OF SCIENCE: Professor Henry Crew, in his presidential address before the American Physical Society,[1] comments unfavorably on the definition of force given by me in a letter in SCIENCE of December 24, 1909, viz., "Force is a pull or a push, something that causes or tends to cause either motion or a change in the velocity or direction of motion." He expresses a "fear" that this definition is used by "not a few students of physics."

An elaboration of the definition, given many years ago by Professor I. P. Church, is as follows:

A force should always mean the pull, pressure, rub, attraction (or repulsion) of one body upon another, and always implies the existence of a simultaneous equal and opposite force exerted by that other body upon the first body, *i. e.*, the *reaction*. In no case should we call anything a force unless we can conceive of it as capable of measurement by a spring balance, and are able to say from what body it comes.

That "a few students of physics" use this definition ought not to be the cause of "fear" to any professor of physics; on the contrary, it should be a source of gratification. It is safe to say that nine tenths of all those students of physics who have occasion after their college days to make use of their physics are going to be either engineers or mechanics, and in that case they will have to learn this "standard definition of the engineer." It is well for them to learn it while they are young.

Professor Crew gives as "the one perfectly *correct, competent* and *complete* description of force" the "rate of change of momentum," and he credits Galileo and Newton with having thus defined it. I can not find, however, in the quotations he gives from Galileo and

[1] SCIENCE, April 8.

Newton any suggestion of such a definition. Galileo, according to the extract quoted, said that "the properties of accelerated motion are defined, without consideration of their causes, in such a way that the momentum (of the body) increases uniformly from the initial condition of rest in simple proportionality to the time," but this is a very different thing from a denial that force causes motion, or from an assertion that force *is* the rate of change of momentum.

Newton's second law of motion, according to one of the translations, is: "If a body be acted on by several forces it will obey each as though the others did not exist, and this whether the body be at rest or in motion." It would be difficult to explain this law if we substitute for the word "forces" the words "rates of change of momentum," especially if the body is at rest and therefore has no momentum.

Referring to the example of the action of force given in my former letter in SCIENCE, a stone is suspended from a projecting shelf by an elastic cord. The earth's gravitation acts on the stone. There is a tension and an elastic resistance in the cord. The word force is used as a generic term to include all those varieties of force that are designated by the words gravitation, tension, resistance, stress, etc. As long as the cord sustains the stone these several forces act, but as there is no motion there is no momentum, nor rate of change of momentum, which Professor Crew says force "is."

Let the cord break. We now have motion, which is change of position during time; velocity, ds/dt; momentum and constant acceleration; all so long as the stone is falling freely, and we may write the equations:

$$FT = MV; \quad F = MV/T; \quad F = MA; \quad V = 2S/T.$$

Before the cord breaks we have two elementary concepts to deal with, matter and force. After the cord breaks, and while the stone is falling, we have two other elementary concepts, space and time, and a few complex concepts: velocity, $V = 2S/T$ or ds/dt, momentum, $W/g \times V$, and acceleration, $(V_2 - V_1)/T$. It is only by a somewhat complex mathematical deduction that we arrive at the "pure concept of the intellect, but a precious concept," $F = (MV_2 - MV_1) \div T$, which Professor Crew says is a "perfectly correct, competent and complete description of force." A boy twelve years old easily grasps the concept that the force acting on the stone is the pull that tends to break the cord, and while he does not know what force is except by its effects, he easily conceives that it is the cause of motion when the cord breaks; it takes a metaphysician to arrive at the definition that force *is* the time rate of the change of momentum.

Let us return to the equations. In order to make them true we must choose certain units for each quantity. Some writers on physics say that the unit of mass is 1 lb. and that the unit of force is a poundal. Others say that the unit of mass is 32.2 lb. and the unit of force 1 lb.; still others that the unit of weight (quantity of matter, W) is 1 lb. and that M is merely an expression to signify W/g. One book on high-school physics defines mass as the quantity of matter, and gives its unit as 1 lb., and also gives the unit of weight (resultant of the attraction of gravitation) as 1 lb., and later gives the equation $W = Mg$, which is wrong if the definitions of the units are right, for in that case $W = Mg$ becomes $1 = 1 \times 32.2$. In the C.G.S. system there is no such trouble, for in it there are four different units to represent the four elementary quantities, viz.: dyne, gramme, centimeter, second. It is only when we try to graft the so-called absolute system on the English system, with its pound representing both quantity of matter and force, and invent new terms, such as the poundal and the gee-pound, to get over the difficulty which exists in the minds of the metaphysical physicists (but not in the minds of engineers, to whom $M = W/g$), that confusion begins.

The equation $F = (MV_2 - MV_1) \div T$ may be interpreted as follows: When a force F acts during a time T on a body which is free to move, and whose mass (W/g) is M, and gives the body an increase of velocity from V_1 to V_2 during that time, then if the units

of the several quantities are chosen so as to make the equation true, the amount of the force is numerically equal to $(MV_2 - MV_1) \div T$, or to the rate of change of momentum.

Let $T = 1$ second, $V_1 = 0$, $V_2 = 32.2$, $W = 1$ lb., $M = 1/32.2$, then the equation reduces to $1 = 1/32.2 \times 32.2$, or force $=$ mass $\times$ acceleration, and it is correct, but if the unit of M is taken as 1 lb. then we have $1 = 1 \times 32.2$, which is incorrect.

The "correct, competent and complete" definition that force *is* the rate of change of momentum, no doubt is a metaphysical deduction from the formula, but it is neither correct, competent nor complete, and is not a definition at all. It assumes that we can translate the sign of equality (=), which really means "is numerically equal to" by the word "is." It is not true even as to equality except under certain limited conditions, viz., 1, that the units have certain values, such as $M =$ lbs. $\div g$, and 2, that the body is free to move. It is not true when a force is applied to a body not free to move, nor when a force is being applied to cause a body to move at a constant speed against a constant resistance, as when a canal boat is being towed, nor when a force is applied to a body moving with increasing speed with decreasing acceleration, as when an engine is bringing a train up to full speed.

"The debt that physics owes to metaphysics" is a sound castigation, for having introduced into physics such bad logic as that of making "equals" equivalent to "is," "darkening counsel with words," and substituting metaphysical deductions and complex concepts for simple definitions and concepts; and for introducing ideas that are so far from being "clear, sharp and definite" that they have to be unlearned or forgotten before the student can make satisfactory progress in engineering mechanics, and that they are discouraging even the high-school physics teachers themselves from teaching elementary dynamics, as was shown in Professor Edwin Hall's paper in Science of October 29, 1909. What is needed is a return to the good old definitions of Weisbach and Rankine, and a dropping of the metaphysical reasoning which has recently become the fashion.

Wm. Kent

Montclair, N. J.,
April 26, 1910

SCIENTIFIC BOOKS

SCIENTIFIC RESULTS OF SHACKLETON'S SOUTH POLAR EXPEDITION

The Heart of the Antarctic: Being the Story of the British Antarctic Expedition, 1907–1909. By E. H. Shackleton, C.V.O. With an Introduction by Hugh Robert Mill, D.Sc. *An Account of the First Journey to the South Magnetic Pole.* By Professor T. W. Edgeworth David, F.R.S. 2 vols., ill., plates. Philadelphia, J. B. Lippincott Co. 1909. $10 net.

It rarely falls to the lot of any single explorer to conjointly arouse such popular interest and contribute such important scientific knowledge as has been done by Sir Ernest H. Shackleton through his Antarctic expedition of 1907–1909. It should be realized that the inception, financeering, organization and successful administration of the expedition are due to Shackleton, it being a private venture unaided, and it may be also said unhampered by governmental offices.

Shackleton played an active part in Scott's Antarctic expedition, 1901–1903—when he was one of the four men who made a world's record of the farthest south—from which he was later invalided on account of scurvy. His early experiences were fruitful factors in his recent successes, which were in a measure due to improved conditions of food, clothing, shelter, transportation and travel methods.

Sailing from New Zealand, January 1, 1908, Shackleton established his permanent station at Cape Royds, Ross Island, at the base of Mt. Erebus. The expedition returned in 1909 with its members in health and its work done with wonderful success. In addition to the meteorological work at Cape Royds, the famous volcano Erebus was ascended and studied, the south magnetic pole located and visited, while the southern party attained a point within 93 geographical miles of the south pole.